AQA Realising potential

2021 ASSESSMENT MATERIALS

AS
MATHS
Algebra and Functions (Topic B)

Model solutions

Total number of marks: 39

1 Identify the expression below that is equivalent to $e^{\frac{-2}{5}}$

Circle your answer.

[1 mark]

$\dfrac{1}{\sqrt[5]{e^2}}$ $\qquad -\sqrt{e^5}$ $\qquad -\sqrt[5]{e^2}$ $\qquad \dfrac{1}{\sqrt{e^5}}$

2 It is given that $y = \dfrac{1}{x}$ and $x < -1$

Determine which statement below fully describes the possible values of y.

Tick (✓) **one** box.

[1 mark]

$-\infty < y < -1$ ☐

$y > -1$ ☐

$-1 < y < 0$ ☑

$y < 0$ ☐

3 It is given that $(x+1)$ and $(x-3)$ are two factors of $f(x)$, where

$$f(x) = px^3 - 3x^2 - 8x + q$$

3 (a) Find the values of p and q.

[3 marks]

$f(-1) = 0 \Rightarrow -p - 3 + 8 + q = 0$ ①

$f(3) = 0 \Rightarrow 27p - 27 - 24 + q = 0$ ②

Simplifying ① and ② we get

① $-p + q = -5$

② $27p + q = 51$

Now we solve simultaniously

② - ① : $28p = 56$

$\Rightarrow p = 2$

So in ① : $-2 + q = -5$

$\Rightarrow q = -3$

3 (b) Fully factorise $f(x)$.

[2 marks]

Our equation is now: $2x^3-3x^2-8x-3$

$(x+1)$ is a factor so we divide by this

$$
\begin{array}{r l}
 & \phantom{2x^3-{}}2x^2-5x-3 \\
x+1 & \overline{\big)\,2x^3-3x^2-8x-3} \\
 & \underline{2x^3+2x^2} \\
 & \quad -5x^2-8x-3 \\
 & \quad \underline{-5x^2-5x} \\
 & \qquad\quad -3x-3 \\
 & \qquad\quad \underline{-3x-3} \\
 & \qquad\qquad\quad 0
\end{array}
$$

So $f(x) = (2x^2-5x-3)(x+1)$

$= (2x+1)(x-3)(x+1)$

4 Show that $\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}}$ can be expressed in the form $m\sqrt{n}+n\sqrt{m}$, where m and n are integers.

Fully justify your answer.

[4 marks]

$$\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} \cdot \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{6}(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$= \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{3-2} = \sqrt{6}\sqrt{3}-\sqrt{6}\sqrt{2}$$

$$= \sqrt{18}+\sqrt{12}$$

$$= 3\sqrt{2}+2\sqrt{3}$$

5 (a) Sketch the curve $y = g(x)$ where

$$g(x) = (x+2)(x-1)^2$$

[3 marks]

<u>Roots</u>

$g(x) = 0$ So $(x+2)(x-1)^2 = 0$

$\Rightarrow x = -2, 1$ (repeated)

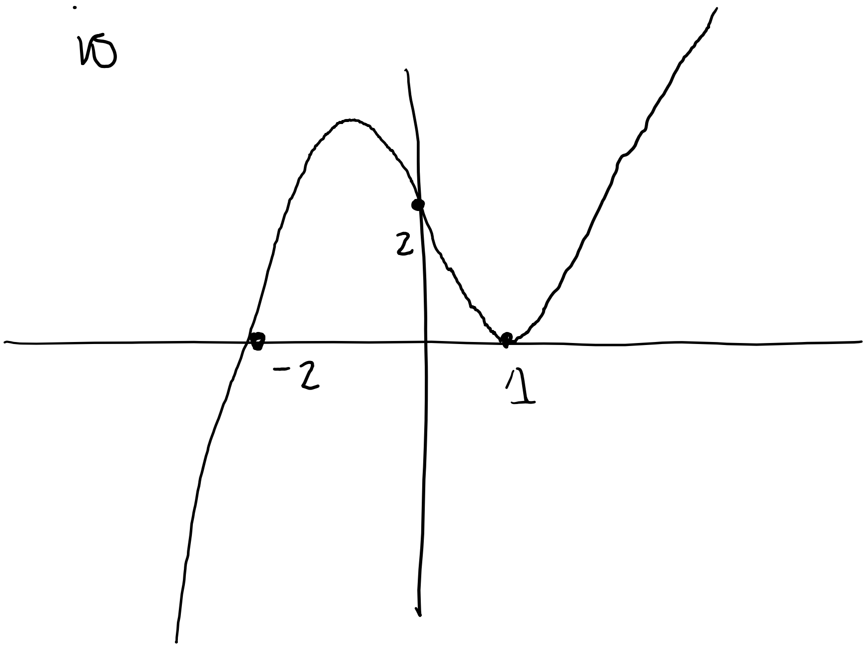
<u>Y intercept</u>

$x = 0$ So $(0+2)(0-1)^2$

$= 2 \cdot -1^2$

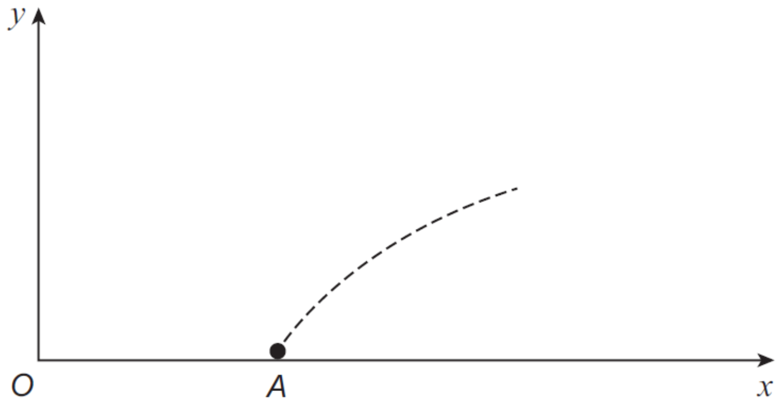
$= 2$

So graph is



11 A fire crew is tackling a grass fire on horizontal ground.

The crew directs a single jet of water which flows continuously from point *A*.



The path of the jet can be modelled by the equation

$$y = -0.0125x^2 + 0.5x - 2.55$$

where x metres is the horizontal distance of the jet from the fire truck at *O* and y metres is the height of the jet above the ground.

The coordinates of point *A* are $(a, 0)$

11 (a) (i) Find the value of a.

[3 marks]

at a : $y = 0$

$$0 = -0.0125x^2 + 0.5x - 2.55$$

$\div -0.0125$ ( ) $\div -0.0125$

$$0 = x^2 - 40x + 204$$

$$(x-34)(x-6) = 0$$

So $x = 6$ as it is the first root

11 (a) (ii) Find the horizontal distance **from *A*** to the point where the jet hits the ground.

[1 mark]

two roots are 34 and 6 so the distance is just the difference

$$34 - 6 = 28$$

distance $= 28$

11 (b) Calculate the maximum vertical height reached by the jet.

[4 marks]

To find the turning point we complete the square

$$y = -0.0125x^2 + 0.5x - 2.55$$

$$= -0.0125\left[x^2 - 40x + 204\right]$$

$$= -0.0125\left[(x-20)^2 - 400 + 204\right]$$

$$= -0.0125\left[(x-20)^2 - 196\right]$$

$$= -0.0125(x-20) + 2.45$$

So the maximum height is 2.45

11 (c) A vertical wall is located 11 metres horizontally from *A* in the direction of the jet. The height of the wall is 2.3 metres.

Using the model, determine whether the jet passes over the wall, stating any necessary modelling assumption.

[3 marks]

A is at $(6,0)$ so the wall is at $(6+11,0) = (17,0)$

Setting $x = 17$ we find the height of the jet at this point:

$$y = -0.0125 \cdot (17)^2 + 0.5 \cdot 17 - 2.55$$

$$\underline{y = 2.3375}$$

So jet passes over the wall

We have assumed air resistance is constant and jet has negligible thickness

7 Given that $y \in \mathbb{R}$, prove that

$$(2+3y)^4 + (2-3y)^4 \geq 32$$

Fully justify your answer.

[6 marks]

$(2+3y)^4$ By Binomial expansion

$$\binom{4}{0} \cdot 2^4 + \binom{4}{1} \cdot 2^3 \cdot 3y + \binom{4}{2} \cdot 2^2 \cdot (3y)^2 + \binom{4}{3} \cdot 2 \cdot (3y)^3 + \binom{4}{4} \cdot (3y)^4$$

$$= 2^4 + 4 \cdot 2^3 \cdot 3y + 6 \cdot 2^2 \cdot 9y^2 + 4 \cdot 2 \cdot 27y^3 + 81y^4$$

$$= 16 + 96y + 216y^2 + 216y^3 + 81y^4$$

$(2-3y)^4$

$$\binom{4}{0} \cdot 2^4 + \binom{4}{1} \cdot 2^3 \cdot -3y + \binom{4}{2} \cdot 2^2 \cdot (-3y)^2 + \binom{4}{3} \cdot 2 \cdot (-3y)^3$$

$$+ \binom{4}{4} \cdot (-3y)^4$$

$$= 16 - 96y + 216y^2 - 216y^3 + 81y^4$$

$81y^4 + 216y^3 + 216y^2 + 96y + 16 + 81y^4 - 216y^3$
$+ 216y^2 - 96y + 16$

$= 162y^4 + 432y^2 + 32$

As $y^4 \geqslant 0$ and $y^2 \geqslant 0$ for all
$y \in \mathbb{R}$

$162y^4 + 432y^2 + 32 \geqslant 32 \quad \forall y \in \mathbb{R}$

$\blacksquare$

7 Curve C has equation $y = x^2$

C is translated by vector $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ to give curve C_1

Line L has equation $y = x$

L is stretched by scale factor 2 parallel to the x-axis to give line L_1

Find the exact distance between the two intersection points of C_1 and L_1

[6 marks]

$y = x^2$ by $\begin{bmatrix} 3 \\ 0 \end{bmatrix} \Rightarrow C_1 : y = (x-3)^2$

$y = x$ by SF 2 parallel to $x \Rightarrow L_1 : y = \frac{1}{2}x$

Finding the points of intersection

$\frac{1}{2}x = (x-3)^2$

$\frac{1}{2}x = x^2 - 6x + 9$

$0 = x^2 - \frac{13}{2}x + 9$

$0 = 2x^2 - 13x + 18$

$0 = (2x-9)(x-2)$

So $x = 2, \frac{9}{2}$

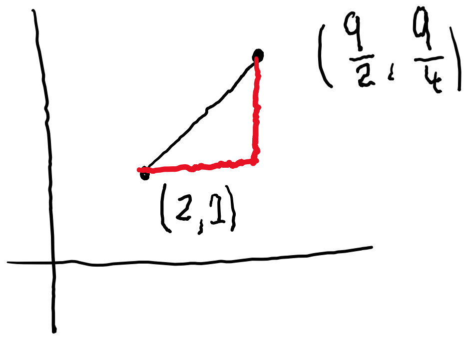
Finding y coordinates

$y = \frac{1}{2}x$

- when $x = 2$ $y = 1$
- when $x = \frac{9}{2}$ $y = \frac{9}{4}$

$(2, 1)$ $\left(\frac{9}{2}, \frac{9}{4}\right)$

Distance between points



Using Pythagorus' theorem

$$\sqrt{\left(\frac{9}{2}-2\right)^2+\left(\frac{9}{4}-1\right)^2}$$

$$= \sqrt{\frac{25}{4}+\frac{25}{16}} = \sqrt{\frac{125}{16}}$$

$$= \frac{\sqrt{125}}{4}$$